

Trade, Employment and the Informal Sector

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The paper corroborates widely observed empirical fact that globalization does increase the size of the informal sector. Liberal trade policy in the form of a decline in tariff reduces open unemployment and increases informal wage and informal employment under reasonable assumption if capital is mobile between the formal and the informal sectors. Liberalization process hurts both organized manufacturing and traditional agriculture and leads to a booming informal sector. Any attempt to artificially raise employment in the formal sector by a wage subsidy leads to higher overall unemployment. Results change drastically when capital is not allowed to be mobile. Thus capital mobility between the formal and informal segments emerges as a critical factor.

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1. Introduction

Whether trade helps or hurts employment has been a topic of never-ending debate among economists, policy makers and activists. Such debate gets even more intense for developing countries experimenting with trade related reforms, since rising unemployment tends to aggravate the problem of poverty and inequality. Typically, when tariffs come down people lose jobs in the import competing sector. As a consequence exportable and total employment can go up. But due to remarkable media coverage of “negative” impact of reforming policies, rising unemployment in the deregulated sectors can create a huge impact on public opinion whereas expanding sectors hardly make the news. There should be a saying that good news is hardly any news but bad news is all that counts. This has a curious resemblance with the well known Gresham’s Law in monetary theory. While total unemployment will be

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determined by the relative size of contraction and expansion, models of trade policy and unemployment have to rely on “ifs and buts” i.e. on specific conditions that affect aggregate unemployment. The dominant wisdom seems to be that more trade is going to hurt import competing sectors all over and thus employment will suffer. In an interesting paper Sarkar (1989) argued how in a Keynesian model trade definitely leads to more unemployment as the sector suffering from effective demand constraints becomes more efficient through trade. But in neo-classical paradigm the impact of trade and unemployment has been an open question.

Variants of Harris-Todaro (HT) structure have been used to look at the impact of trade policies on open unemployment and welfare. Interested readers may look at Beladi and Marjit (1996), Marjit and Beladi (2003), and Fields (2005).¹ We relate our paper to a particular strand of HT literature that deals with the informal sector. Gupta (1997) introduced informal sectors in a HT model. But the urban informal wage in such models is less than the rural wage, as expected urban wage has to be equal to the rural wage and the urban formal wage is greater than the rural wage. Casual empiricism suggests that in the developing world there is a fair bit of mobility between the urban informal sector and the rural sector. In our framework workers get the same wage in both sectors. The urban informal sector draws labor from the rural sector and capital from the urban formal sector. Such mobility assigns a pivotal role to the informal sector. The way we set up the model allows workers the choice of either working in the informal sector or going back to the rural sector and of course the choice of “waiting” in the pool of unemployed. One may imagine the urban informal sector to be located in the periphery of the city, between the formal and the rural sector.

Empirically the significance of the urban informal sector has been understood in many papers. Agenor (1996) provides an elegant survey on the size of informal sectors in developing countries. By quoting numerous studies, informal labor force accounts for more than 70% of total labor force. Segmented labor markets and implications of development policies in such a set up have been analyzed in Agenor and Montiel (1996). Drawing on earlier papers by Agenor and Montiel (1996), Marjit (2003), Agenor (2005) and Marjit, Kar and Beladi (2007) looked at the possibility of a rising informal wage and employment when laid off workers from the formal sector crowd into the informal sector. But somehow employment effect of trade policy in a standard HT structure with an informal sector is largely absent in the literature.

It is difficult to obtain data on informal sectors in any country. However, there is some empirical evidence which shows that during the recent trend in globalization, developing countries are experiencing an increase in the size of the informal economy. Also Marjit and Maiti (2006) demonstrate with the figures available for the informal manufacturing sector in India before and after the initiation of the reform process, that the informal wage and employment have increased across various Indian states during the post reform period, at least up to the last but one round of the NSS. It has tapered of a bit in the most recent period. It also shows the growth of fixed assets in the informal segment when the rate of capital

¹ For related studies on HT Model see Khan (1982), Gupta (1993, 1997), Oladi (2004), Chao and Yu (1995, 2001), Chakrabarti (2004) among others.

accumulation in formal manufacturing was fairly insignificant. In fact a careful look at the surging GDP growth rate in recent times will reveal that aggregate rate of investment in manufacturing, as available from ASI figures, has not taken off commendably. In fact it is the service sector which has been fuelling most of India's growth. However, unorganized manufacturing possibly tells us a different story. In our framework a decline in tariffs reduces open unemployment through expansion of employment in the informal sector and an increase in the informal wage. The result is an outcome of having a labor intensive urban informal sector and allowing capital to move between the formal and the informal sector. It is more simple and general than the earlier work of Beladi and Marjit (1996) which brings in an intermediate input in the HT structure to get the employment reducing effect of protection. We also demonstrate that our structure naturally leads to the well known Todaro-paradox. Any attempt to raise employment through wage subsidy in fact leads to an increase in overall unemployment. Capital mobility plays a very crucial role in our analyses as it has done in some others in related literature. If capital does not move at all between the formal and the informal sectors, the results will change drastically.

The plan of the paper is as follows. Second section discusses the model and the equilibrium. The third section deals with the impact of a tariff when the informal is a traded sector and the impact of a wage subsidy. The fourth extends the result by treating the informal good as the non-traded good. The last section provides some concluding remarks and discusses the case with zero or restricted mobility of capital between the formal and the informal sectors.

2. The Basic Framework and Equilibrium

This is a three sector economy producing X (urban formal manufacturing good), Y (urban informal good), and Z (the agricultural good). X and Y use capital and labor, Z uses land and labor. Labor is freely mobile between Y and Z , in the sense that workers earn the same wage, W , informal in both these sectors. But X offers a wage \bar{W} , formal wage determined through negotiation with the unions. There is open unemployment i.e. the workers migrating from the rural to the urban area can either hang around unemployed waiting for a formal job which pays $\bar{W} > W$ or they can get W in the urban informal sector or in the rural sector. They are indifferent between the two. Capital moves freely between the urban formal and informal sectors earning the same return, r .

We assume that X is an import-competing good enjoying a tariff, t . On the other hand, Y , the informal manufacturing sector is assumed to produce a traded good in the benchmark model. Later we introduce the possibility that Y is a non-traded good. The agricultural good, Z is an export good. Competitive markets and constant returns to scale technology with diminishing marginal productivities of factors are assumed. Competitive price conditions imply:

$$\bar{W}a_{LX} + ra_{KX} = P_X(1+t) \quad (1)$$

$$Wa_{LY} + ra_{KY} = P_Y \quad (2)$$

$$Wa_{LZ} + Ra_{TZ} = P_Z \quad (3)$$

a_{ij} 's are rural input-output coefficients, r is the return to capital and R is rental on land.

Full employment of land and capital ensure that,

$$a_{TZ}Z = T \quad (4)$$

$$a_{KX}X + a_{KY}Y = K \quad (5)$$

where T and K are inelastic supplies of land and capital. Harris – Tadaro migration equilibrium condition is given by,

$$\frac{\bar{W}a_{LX}X}{L - (a_{LY}Y + a_{LZ}Z)} = W \quad (6)$$

Note that (6) can be rewritten as a modified “full-employment” condition for labor.

$$\frac{\bar{W}}{W}a_{LX}X + a_{LY}Y = L - a_{LZ}Z \quad (7)$$

With given prices and tariff rate, t , one can determine W , r , and R from (1) – (3). Factor prices in turn determine factor proportion. Now from (4) we can determine Z . Then (5) and (7) determine X and Y . This completes the determination of equilibrium. We assume that Y is labor intensive and X is capital intensive, an assumption which hardly needs any justification. However, the factor intensity assumption implies,

$$\left[\frac{\bar{W}}{W}a_{LX}a_{KY} - a_{KX}a_{LY} \right] < 0$$

or,
$$\frac{a_{KX}}{a_{LX}} > \frac{\bar{W}}{W} \bullet \frac{a_{KY}}{a_{LY}} \quad (8)$$

Note that on, $\bar{W} > W$, capital intensity assumption is a bit stronger than the usual i.e.

$$\frac{a_{KX}}{a_{LX}} > \frac{a_{KY}}{a_{LY}} \quad (9)$$

Let us define (8) as the stronger version of intensity assumption and the weaker version is given by,

$$\left(\frac{\bar{W}}{W}\right) \bullet \left(\frac{a_{KY}}{a_{LY}}\right) > \frac{a_{KX}}{a_{LX}} > \frac{a_{KY}}{a_{LY}} \quad (10)$$

Under the weaker version, the informal sector becomes effectively the capital-intensive sector. (10) Accommodates a kind of factor intensity reversal if W goes down way below \bar{W} i.e. wider is the formal wage gap, we may have a factor intensity reversal. Henceforth, we shall assume (8) holds and assume away any possible factor intensity reversal.

3. A Reduction in tariffs

Let us trace through the changes in equilibrium as t is reduced. A decline in t reduces r , increases W and reduces R . This follows directly from equations (1) – (3). A decline in R makes people use land more intensively and with a given T that reduces Z [from (4)] and a_{LZ} . As the RHS in (7) increases with a decline in $a_{LZ}Z$, the LHS reduces with a decline in (\bar{W}/W) . Also (a_{KX}, a_{KY}) increases with an increase in (\bar{W}/r) and (W/r) . Hence, the labor constraint [(7)] becomes less binding and the capital constraint [(5)] becomes more binding, This leads to the well known Rybczynski type result increasing the labor-intensive Y (urban informal sector) and contracts the capital intensive X (urban formal manufacturing good).

Now from (7) we can write total employment, L_E , as

$$\begin{aligned} L_E &\equiv a_{LX}X + a_{LY}Y + a_{LZ}Z = L - \frac{\bar{W}}{W}a_{LX} + a_{LX}X \\ &= L - a_{LX}X \left[\frac{\bar{W}}{W} - 1 \right] \end{aligned} \quad (11)$$

By now we know that $a_{LX}X$ must have gone down and (\bar{W}/W) also has gone down. Therefore, L_E must increase. We can now write the following proposition.

Proposition I: *Under the stronger factor intensity ranking a decline in tariffs must increase total employment.*

The exact proof is given in the appendix. Note that a downsized urban formal sector, with an expanding urban informal sector and a rising informal wage must reduce open unemployment in the economy. A decline in tariffs not only contracts formal manufacturing, but also traditional agriculture. The existing set of informal and rural workers must gain as W is higher now. Displaced workers from the formal sector lose as they were getting \bar{W} . Some of those who were in the pool of “open unemployed” give up hope and join the informal sector.

Typically a drop in t will reduce the expected wage rate in the urban formal sector and has a tendency to reduce open unemployment. But in the standard HT framework people migrating to agriculture depresses the wage rate in agriculture. Therefore, the effect on expected wage

rates is ambiguous, so is for unemployment. However, in our framework W in fact goes up as displaced capital from the formal sector goes to the informal sector. This is a must when there is not much of a change in P_Y . Here a drop in t unambiguously reduces total unemployment or increases total employment. It is the rise in the informal wage which is a striking outcome because the conventional wisdom will be a drop in W as more people crowd into the informal segment. But as capital leaves the formal sector and as r goes down, capital-labor ratios in each sector go up, driving up W . Not only the displaced workers from the formal sector are absorbed in Y but higher W also attracts more agricultural workers to the informal sector raising overall employment. Also note that the average wage which is a weighted average of formal wage \bar{W} and W is given by,

$$\begin{aligned}
 W_a &= \bar{W} \cdot \frac{L_x}{L} + \frac{W(L_Y + L_Z)}{L} \\
 &= \bar{W} \cdot \frac{L_x}{L} + W \left[\frac{L - L_x \left(\frac{\bar{W}}{W} - 1 \right) - L_x}{L} \right] \quad (\text{from (4)}) \\
 &= W + \frac{L_x}{L} \left[\bar{W} - W \left(\frac{\bar{W}}{W} - 1 \right) - W \right] \\
 &= W + \frac{L_x}{L} [\bar{W} - \bar{W} + W - W] \\
 &= W
 \end{aligned}$$

Clearly, the average wage is nothing but W itself. Hence, the average worker must gain from a tariff reform. Therefore, a rise in W must mean a rise in W_a and hence the welfare level of an average worker.

4. The Non-Traded Informal Sector

If the urban informal good, Y is non-traded, we need a separate equation for determining the equilibrium P_Y by balancing demand and supply in this sector. Let us look at the sequence of outcomes following a decline in tariff in such a context.

In our earlier analysis Y definitely expands due to the assumption of factor intensity ranking between X and Y . Now an increase in Y reduces P_Y to clear the market for the informal good and a decline in P_Y tends to reduce W , offsetting the positive effect on employment.

However, a decline in t and a decline in P_Y have opposite effects on demand for Y . What we have shown in the appendix [(A.13)] suggests that a high elasticity of substitution in demand (μ_2) will be a sufficient condition to reinstate our earlier result. In fact, it is well known that if $\mu_2 \rightarrow \infty, \hat{P}_Y \rightarrow 0$ i.e. in the limit it mimics the case with a traded informal

sector. More generally, even if W goes down, strong elasticity of factor substitution in X , σ_X , will mean an increase in employment. But there is every possibility that the informal wage will go up.

One has to realize that a high σ_X , reduces $(a_{LX} X)$ significantly and workers with depressed wages do not hang around in the hope of a job and turn back to Y , increasing employment.

Proposition II: *If Y is the urban non-traded sector, a decline in tariffs will increase total employment provided that σ_X and μ_2 are strong enough.*

For proof of this proposition, see the derivation of (A.13) in the appendix.

5. Related Discussion

It is well known in the HT framework that employment and aggregate labor income may move in opposite directions following a policy change. Aggregate labor income in our model is given by.

$$\bar{W}a_{LX}X + Wa_{LY}Y + Wa_{LZ}Z = WL \quad (12)$$

Whatever happens to employment, aggregate labor income increases with an increase in W , the informal/rural wage. What we have shown here is that a decline in t can simultaneously increase employment and raise W , provided the condition (A.13) holds. If P_Y does not move much, W must increase. If P_Y does change a lot, W can still go up [see (A.10) in the appendix]. The weaker factor-intensity ranking condition,

$$\frac{\bar{W}}{W} \bullet \frac{a_{KY}}{a_{LY}} > \frac{a_{KX}}{a_{LX}}$$

leads to a capital intensive informal sector, as a substantial wage gap (\bar{W}/W) makes worker's share of average cost in X quite high relative to the informal sector. Therefore, a drop in t may actually increase X and reduce Y . Yet L_E may very well increase since a rise in W will always have a positive effect on total employment by cutting back open unemployment. As there is a general consensus that the informal sector is labor intensive compared to formal sectors, we prefer to retain the stronger factor intensity assumption as preserved usual factor intensity ranking.

Another point to note in this context is that a rise in the effective price of Z , may be through an increase in prospects for exports or through an increase in productivity is going to increase unemployment. A rise in P_Z does not change W . But an increase in Z reduces total labor available for the production of X and Y . Since X is capital-intensive, due to the Rybczynski effect, X goes up and L_E goes down. Informal sector contracts but X expands,

more people wish to hang around for the job in the formal sector, increasing the extent of open unemployment.

One issue that seems to be left open is the pattern of trade. As t goes down both X and Z contract. This means that import-competing output and export production both decline. A natural query is how to sustain balance of trade. Of course as $P_x(1+t)$ and P_y drop relative to that of Z , exportable surplus can very well increase. If not, income effect will make sure that consumption adjusts to balance of trade constraint. But a simple extension of the basic model will be much more realistic.

Consider a situation where we also have a sector which uses skilled labor and capital and produces an exportable. A drop in t , by reducing r will increase skilled wage and skilled output of the exportable. This can be added without altering any of our basic results. Skilled labor, just as capital, can be assumed to be a scarce input and fully employed. One additional interesting result that one may obtain in the extended model has to do with the worsening of unskilled/skilled wage gap following a drop in t if capital's share in average cost is greater in the skilled sector than the unskilled sector.

Another interesting result of this paper has to do with the policy of subsidizing employment in the urban formal sector. In the traditional Harris – Todaro literature it was demonstrated that the effect of a wage subsidy can lead to an increase in unemployment. Basu (1977) discusses this in great detail, although the paradox does get resolved when agriculture is decomposed into two sub-sectors and urban wage subsidy helps the labor intensive segment. [Marjit (1991)]. In the present paper any attempt to raise formal sector employment by artificial wage subsidy *must* lead to *greater unemployment* on the whole. The mechanism is exactly symmetric to the one discussed earlier.

A wage subsidy must raise return to capital, r . That in turn reduces the informal and rural wage rate. As capital constraint becomes less binding and the labor constraint gets harder, X must expand and Y must contract. From (11) it is straightforward to argue that total employment in X will increase and relative urban wage will also increase reducing the aggregate employment.

6. Concluding Remarks

We address one of the most controversial issues in trade policy of the developing nations. Does trade increase employment?

We argue that a decline in tariff helps create jobs in the informal sector and increase informal wage at the same time even when workers are laid off from the formal sector. This may accompany a decline in rural employment. But overall employment must increase.

The pivotal role of an urban informal sector is justified by the fact that if we did not have the informal sector in our model we could not have obtained a totally different result. A drop in

t will reduce urban employment and reduce W , the rural wage rate. Although there will be offsetting effects on open unemployment, rural workers would have been definitely worse off and unemployment could easily go up. It is the informal sector which draws in capital, preserves and raises employment and also allows the informal wage to increase. Even if people leave rural areas, open unemployment is not allowed to grow.

Our results are sensitive to the assumption of capital mobility. Suppose we have absolutely no mobility of capital. In that case the endowment and price equations are not separable and we need the full employment conditions for both types of capital, land and migration equation to solve for factor prices. Then the system has to be solved simultaneously. Now a drop in t , reduces employment in X and capital can not move out, so w must go down. And aggregate unemployment must increase. However, one can show some degree of mobility will preserve the result and we do not necessarily need perfect mobility of capital. We need to release capital from X to the extent that K/L ratio in Y goes up even if some labor is released from X . One can model the adjustment by defining a critical level of elasticity of mobility of capital beyond which w will actually increase.

It will be easy to graft a sector that produces skilled manufacturing good or service by using skilled labor and capital, onto our framework. Nothing will change except that this sector, presumably an export sector, will also benefit from a cut back in tariff as it has to pay a lower return to capital. In that case several comparative static experiments will yield results where the skilled sector and the informal sector will demonstrate a degree of complementarity in production. A general theoretical discussion on such issues is available in Marjit (2005).

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Appendix

Effect of a change in tariff (t):

Using (4) and (7) we derive (A.1)

$$\frac{\bar{W}}{W} a_{LX} X + a_{LY} Y = L - \frac{a_{LZ}}{a_{TZ}} \bullet T \quad (\text{A.1})$$

$$a_{KX} X + a_{KY} Y = K \quad (\text{A.2})$$

Differentiating and using ‘^’ to denote proportional change we get,

$$\lambda_{LX} \hat{X} + \lambda_{LY} \hat{Y} = \delta_L \hat{W} \quad (\text{A.3})$$

$$\lambda_{KX} \hat{X} + \lambda_{KY} \hat{Y} = -\delta_K \hat{W} \quad (\text{A.4})$$

$$(\delta_L, \delta_K) > 0$$

Derivation here follows Beladi and Marjit (1996). Now from competitive price conditions we get,

$$\hat{r} = \frac{\hat{\tau}}{\theta_{KX}}$$

where, $\tau \equiv (1 + t)$

$$\hat{W} = \frac{\hat{P}_Y - \frac{\theta_{KY}}{\theta_{KX}} \bullet \hat{\tau}}{\theta_{LY}} \quad (\text{A.5})$$

$$\hat{W} > 0 \text{ as } \hat{\tau} < 0$$

Therefore we have,

$$\hat{X} = \frac{\begin{vmatrix} \delta_L \hat{W} & \lambda_{LY} \\ -\delta_K \hat{W} & \lambda_{KY} \end{vmatrix}}{|\lambda|} = \frac{(\delta_L \lambda_{KY} + \delta_K \lambda_{LY}) \hat{W}}{|\lambda|}$$

where,

$$|\lambda| \equiv \left[\frac{\hat{W}}{W} a_{LX} a_{KY} - a_{KX} a_{LY} \right] < 0$$

(by stronger factor intensity assumption), hence

$$\hat{Y} = \frac{\begin{vmatrix} \lambda_{LX} & \delta_L \hat{W} \\ \lambda_{KX} & -\delta_K \hat{W} \end{vmatrix}}{|\lambda|} = \frac{-\hat{W}}{|\lambda|} [\lambda_{LX} \delta_K + \lambda_{KX} \delta_L] \equiv \hat{W} \bullet A, A > 0$$

So that,

$$\hat{Y} = \left[\frac{\hat{P}_Y - \frac{\theta_{KY}}{\theta_{KX}} \hat{\tau}}{\theta_{LY}} \right] \bullet A = \frac{\hat{P}_Y}{\theta_{LY}} A + \frac{\theta_{KY}}{\theta_{KX} \theta_{LY}} (-\hat{\tau}) \bullet A \quad (\text{A.6})$$

Note that when Y is a traded good and P_Y is given, $\hat{X} < 0, \hat{Y} > 0$. If Y is a non-traded good, demanded for Y and supply of Y must match locally. Let us postulate a demand function for Y .

$$\hat{Y}_D = \mu_1 \hat{\tau} - \mu_2 \hat{P}_Y \quad (\text{A.7})$$

A change in τ has both income and substitution effects on demand for Y . While the substitution effect is negative i.e. a drop in τ reduces Y_D , the income effect can go either way. μ_1 captures both. μ_2 is own price effect and in general equilibrium any changes in P_y will have income effect as well. We will assume their substitution effect dominates and μ_1, μ_2 are positive

Equating (A.6) and (A.7) we get the equilibrating change in P_Y as,

$$\frac{\hat{P}_Y}{\theta_{LY}} \bullet A + \frac{\theta_{KY}}{\theta_{KX} \theta_{LY}} (-\hat{\tau}) A = \mu_1 \hat{\tau} - \mu_2 \hat{P}_Y \quad (\text{A.8})$$

or,

$$\hat{P}_Y = \left\{ \frac{\hat{\tau} \left[\mu_1 + \frac{\theta_{KY} A}{\theta_{KX} \theta_{LY}} \right]}{\mu_2 + \frac{A}{\theta_{LY}}} \right\} < 0 \quad (\text{A.9})$$

Therefore,

$$\begin{aligned}
 \hat{W} &= \hat{\tau} \frac{\mu_1 \theta_{KX} \theta_{LY} + \theta_{KY} A}{(\mu_2 \theta_{LY} + A) \theta_{KX} \theta_{LY}} - \frac{\hat{\tau} \theta_{KY}}{\theta_{KX} \theta_{LY}} \\
 &= \frac{-\hat{\tau}}{\theta_{KX} \theta_{LY}} \left[\theta_{KY} - \frac{\mu_1 \theta_{KX} \theta_{LY} + \theta_{KY} A}{\mu_2 \theta_{LY} + A} \right] \\
 &= \frac{-\hat{\tau}}{\theta_{KX} \theta_{LY}} \left[\frac{\theta_{LY} [\theta_{KY} \mu_2 - \theta_{KX} \mu_1]}{\mu_2 \theta_{LY} + A} \right]
 \end{aligned} \tag{A.10}$$

Therefore, $\hat{W} > 0$ if

$$\mu_2 > \frac{\theta_{KX}}{\theta_{KY}} \bullet \mu_1$$

We now look at the effect of a decline in t on the total unemployment L_E .

$$L_E = a_{LX} X + a_{LY} Y + a_{LZ} Z = \bar{L} - a_{LX} X \left(\frac{\bar{W}}{W} - 1 \right) \tag{A.11}$$

$$\hat{L}_E > 0 \text{ iff } \left[a_{LX} X + \left(\frac{\bar{W}}{W} - 1 \right) \right] < 0$$

$$\hat{L}_E > 0 \text{ iff } \left\{ \left[-\theta_{KX} \sigma_X (-\hat{\tau}) + \hat{W} \bullet B \right] - \hat{W} \frac{\frac{\bar{W}}{W}}{\left[\frac{\bar{W}}{W} - 1 \right]} \right\} < 0$$

where, $B = \left[\frac{\delta_L \lambda_{KY} + \delta_K \lambda_{LY}}{|\lambda|} \right] < 0$.

Hence, $L_E > 0$ iff $\sigma_X \bullet \hat{\tau} + \hat{W} \left[B - \frac{\frac{\bar{W}}{W}}{\frac{\bar{W}}{W} - 1} \right] < 0$ (A.12)

It is now readily available from (A.12) that since $\hat{\tau} < 0$ and $B < 0$, $\hat{L}_E > 0$ if $\hat{W} > 0$.

We already noted that for $\hat{P}_Y = 0$, $\hat{W} > 0$ as $\hat{\tau} < 0$ [from (A.5)]. Therefore, L_E must go up when τ goes down – the main proposition of the paper. The general expression for (A.12) is given by (after substitution for \hat{W}).

$$\sigma_X \hat{t} - \hat{t} \left[\frac{\theta_{KY} \mu_2 - \theta_{KX} \mu_1}{\theta_{KX} (\mu_2 \theta_{LY} + A)} \right] \left[B - \frac{\bar{W}}{W} \right] = \hat{t} \left[\sigma_X - \frac{\theta_{KY} \mu_2 - \theta_{KX} \mu_1}{\theta_{KX} (\mu_2 \theta_{LY} + A)} \left(B - \frac{\bar{W}}{W} \right) \right]$$

Therefore as $\hat{t} < 0$, $\hat{L}_E > 0$ iff,

$$\sigma_X > \frac{\theta_{KY} \mu_2 - \theta_{KX} \mu_1}{\theta_{KX} (\mu_2 \theta_{LY} + A)} \left(B - \frac{\bar{W}}{W} \right) \quad (\text{A.13})$$

Since the bracketed term in RHS of (A.13) is negative, a sufficient condition for an increase in employment following a decline in t is given by,

$$\frac{\theta_{KY} \mu_2 - \theta_{KX} \mu_1}{\theta_{KX} (\mu_2 \theta_{LY} + A)} > 0$$

which is also the condition for an increase in W .