The Leapfrogging Model of Income Leadership in International Trade with Specified Learning

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The Leapfrogging model of income leadership in trade gives an economic explanation as why open economies overtake, e.g. leapfrog, each other in per capita income. This Leapfrogging mechanism is based on endogenous learning and exogenous technological change. Albeit, it is carefully described in literature, it lags the specification, e.g. when does Leapfrogging really occur. In this paper we specify learning with a simple yield curve. With this specification, we can prove whether Leapfrogging can take place or not.

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1. Introduction

It is often seen by ranking per capita income of countries/regions over time, that economies change leadership positions. Such an example is illustrated in Figure 1, which shows per capita income of the UK relative to the Netherlands over more than 150 years.

The evolution of per capita income leadership and the reversals in this example cannot simply be described by convergence or divergence approaches (c.f. Barro and Sala-i-Martin, 1992, Aghion and Howitt, 1998 respectively).

To explain such overtaking and being overtaken, Brezis, Krugman and Tsiddon (1993) coined the term Leapfrogging and created a simple but impressive model. In a 2x2 model, the economy with a higher expertise in a particular sector becomes the income leader. It can maintain its productivity advantage because its specialisation in trade also gives the economy the best chance to improve the level of experience through endogenous learning by doing. But income leadership is always susceptible to exogenous fundamental technological change.
Such a change may reverse fortunes because the more experienced economy has higher opportunity costs to switch to new technologies. That is the reason for new fundamental methods being introduced in backward places, where the implementation should immediately pay off. Furthermore, learning in new methods is faster than in older ones which have already been improved by past experiences. So the ‘once backward economy’ may overtake productivity leadership in the particular sector, and as a result may become income leader.

But the last part of this story remains intuitive in Brezis et al. (1993). Without specifying learning dynamics, these authors just assume that it is going to happen. This paper is closing that gap in literature by the implementation of a simple yield curve of learning in the model. With this extension, a threshold emerges which bounds the Leapfrogging to the relative technological improvement through the new method, to the expenditure share of the particular sectors products and to the learning dynamics in this sector.

The paper is organised as follows. The second part explains in short the Leapfrogging model. The third part introduces a simple yield curve into the model and establishes the necessary dynamic condition for Leapfrogging. The fourth part concludes.

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1The First and the Second World War periods are excluded.
2. The Leapfrogging Model

2.1. Model setting

The Leapfrogging model is a Ricardian model in which two countries Home and Foreign (\( \dagger \)), trade two goods, food (\( F \)) and manufactures (\( M \)). Both goods can be produced in both countries with the one factor input, \( L \) and \( L' \), respectively. Both countries are equal sized, so \( L = L' \).

Production of food is subject to identical, normalized technologies in both countries:
\[
F = L_F \quad \text{and} \quad F^* = L_F^* .
\]

Agriculture technology remains static over time. Production of manufactures is at the beginning also subject to the same technology generation one (\( A_1 \)) in both countries:
\[
M_1(T) = A_1(K_1(T))L_M , \quad \text{and} \quad M_1^*(T) = A_1^*(K_1^*(T))L_M^*. \tag{2}
\]

Manufacturing productivities \( A_i \) and \( A_i^* \) are functions of accumulated output \( K_1 \) and \( K_1^* \) in Home and in Foreign, respectively and are assumed to be strictly increasing and concave:
\[
A_i' = dA_1(K_1(T))/dK_1 > 0 , \quad A_i^* = dA_1^*(K_1^*(T))/d^2K_1 < 0 , \tag{3}
\]

Output in manufacturing is accumulated in Home and in Foreign over the past till time \( T \):
\[
K_1(T) = \int_{-\infty}^{T} M_1(t)dt , \quad \text{and} \quad K_1^*(T) = \int_{-\infty}^{T} M_1^*(t)dt . \tag{4}
\]

Thus, productivity in manufacturing increases over time endogenously by learning by doing. This effect is country-specific. Now assume that Foreign has had a higher manufacturing output in the past, and thus manufacturing productivity in Foreign is higher than in Home, i.e., \( A_i'(K_1^*(T)) > A_i(K_1(T)) \), which gives Foreign the absolute and the comparative advantage over Home in this sector.

Finally, we assume an identical Cobb-Douglas-utility function, i.e., expenditure share preferences, in each country is given by:
\[
U = D_M^\mu D_F^{1-\mu} . \tag{5}
\]

2.2. Resulting Production Patterns in Trade

The Ricardian model of this type gives three possible production patterns. As explained further in Brezis et al. (1993), wage inequality only occurs, with a higher expenditure share for
manufactures $0.5 < \mu < 1$. This excludes the pattern in which only Home specialises in agriculture. So, the model allows only the production pattern in which both countries specialise, or in which only Foreign can specialise in manufacturing.

Both production patterns are distinguishable through a consideration of relative wages. Relative wage in specialisation with normalized inputs reflects world expenditure shares for food and manufactures:

$$\frac{w}{w^*} = \frac{1 - \mu}{\mu}.$$  \hfill (6)

Relative wage with manufacturing in both countries reflect only productivity differences in this sector:

$$\frac{w}{w^*} = \frac{A_1(K_1(T))}{A^*(K^*_1(T))} \hfill (7)$$

Home manufacturing only exists, if productivity differences do not grow too large:

$$\frac{1 - \mu}{\mu} < \frac{A_1(K_1(T))}{A^*(K^*_1(T))} \hfill (8)$$

Distinctions between these productivity patterns are important for the following necessary conditions of leapfrogging.

2.3. Technological Change in Manufacturing and Leapfrogging

Leapfrogging can only occur in this model with exogenous technological change in the manufacturing sector which alters the disadvantage of lower Home learning by doing in $A_1$. Brezis et al. (1993) assume here a technology shock, which provides a better technology generation two ($A_2$) at time $T$, at an initial no experience level, $A_2 > A_1$, and which does not allow any spillover of learning by doing from $A_1$ to $A_2$.

This new technology is useful for Home to leapfrog Foreign only if Home can apply the new technology in manufacturing and Foreign cannot. If the technology is not applicable at Home, it cannot be applied in Foreign as well. If this technology is applied in Foreign, the productivity and income differentials vanish. Brezis, et al. (1993) explain further, why in perfect competition manufacturers may not be able to overcome an initial productivity advantage.
So given a production pattern of specialisation in Foreign and relative specialisation in Home with $A_1$, $A_2$ can be implemented in Home only, if its initial productivity is at least equal with the former one but less productive in foreign. So the technology shock in $T_1$ has to be characterized by:

$$A_1 \left(K_1(T_1)\right) \leq A_2 < A_1^* \left(K_1^*(T_1)\right) .$$

(9)

To implement the new technology at Home from a previous production pattern of specialisation demands a change in minimum productivity at Home because it has to go along with a change in this production pattern. If, with $A_1$, Home was not able to fulfil condition (8), it has to be fulfilled with $A_2$. So the technology shock in $T_1$ from specialisation has to be characterized by:

$$A_1^* \left(K_1^*(T_1)\right) \frac{1-\mu}{\mu} < A_2 < A_1^* \left(K_1^*(T_1)\right)$$

(10)

With equations (9) and (10), Brezis et al. (1993) gives the necessary conditions to implement a new technology solely in the backward country related to the previous Ricardian production pattern. Using the new technology at Home manufacturing and the previous one in the Foreign sector Home productivity may increase more quickly because of faster learning by doing in an immature technology. And with faster learning by doing, Home manufacturing productivity may overtake Foreign because of the better technology generation two. Thus, (9) and (10) are necessary conditions for the exogenous technology shock to start Leapfrogging. The repetition of such technological shocks creates a cycle of backwardness and prosperity in production and income. Brezis et al. (1993) end here.

The problem with (9) and (10) is that it can only fix the possible starting point of Leapfrogging in a static sense but it cannot define dynamic Leapfrogging. The exclusive implementation of the new technology in Home does not imply that Home overtakes Foreign over time. This is because the learning by doing with it should not to be faster than in Foreign as Home has disadvantageous relative specialisation in agriculture. And also with initially higher learning dynamics this should not persist until Foreign has to be overtaken. Along with this two necessary static conditions chapter 3 also introduces an additional dynamic condition. The dynamic condition distinguishes implemented new technologies which guarantee Leapfrogging from others which do not. For this purpose, it is necessary to specify the model further by introducing a simple yield curve of learning.
3. Productivity dynamics and the necessary condition for dynamic Leapfrogging

3.1. Implementing a simple yield curve of learning

Productivity dynamics through learning by doing are only imprecisely defined by Brezis et al. (1993) with conditions (3). To realise the exact dynamics we specify here a simple yield curve of learning. Therefore, we model productivity in $A_t$, as the product of the initial technology specific productivity $A_1$ and the accumulated output $K_1(T)$ at $T$ over the elasticity $b$:

$$A_t(K_1(T)) = A_1K_1(T)^b$$  \hspace{1cm} (11)

This simple yield curve is strictly concave an increasing with $0 < b < 1$ and an initial stock of output at the technology shock of one unit, $K_1 = 1$. A technology shock makes available a $A_2$ which has a higher initial productivity than the previous one $A_2 > A_1$.

The chosen yield curve is a specific formulation to define the productivity dynamics and the Leapfrogging properly. Also probably it’s the easiest way to introduce productivity dynamics into the model, as described with the so-called “learning curve” in theory and empirics (c.f. Argote and Epple, 1990). Equation (11) is close proxy to the learning curve, so the elasticity $b$ can be described as the so-called “learning elasticity”, and the initial productivity $A_1$ is the reciprocal value of the labour input coefficient, which characterises the initial direct labour cost in units of labour.

3.2. Productivity dynamics at Home with the new technology and in Foreign with the previous technology

Productivity dynamics are derived from (2) and (4) in the general form as:

$$\dot{A}_t = A'_tM_t\left(K_1(T)\right) = A'_tA_1(K_1(T))L_M$$  \hspace{1cm} (12)

We know for Foreign, that its productivity dynamics that occurs after the technology shock and before Leapfrogging, are characterized by the use of previous technology in a production pattern of specialisation in manufacturing, $L^* = L^*_{LM} = 1$. Using this information together with the simple yield curve from (11) we get:

$$\dot{A}_t = bA_1^2\frac{K_1^*(T)^{2b}}{K_1(T)}$$  \hspace{1cm} (13)
For Home, we know that it has adopted $A_2$ with the technology shock in its manufacturing sector. Furthermore, before having leapfrogged Foreign, Home is relatively specialised in agriculture so that $L_M$ is smaller than $L$.

We derive $L_M$ from world income $Y$ with partial specialisation at Home and full specialisation at Foreign. For normalized Home wage and normalized labour in both countries world income is:

$$Y = 1 + w^*/w$$

(14a)

Furthermore, expenditure share considerations give us the fraction of food expenditure $(1 - \mu)Y$, which has to be equal with wage of Home labour in agriculture:

$$(1 - \mu)Y = L_F.$$  

(14b)

From (14), we can derive Home labour in agriculture as

$$L_M = \mu - (1 - \mu)\frac{A_1^*}{A_2} \left( \frac{K_1(T)}{K_2(T)} \right)$$

(15)

Productivity dynamics at Home are determined by implementing (15) into (12) and still using of the yield curve (from 11):

$$A_2 = bA_2^2 \frac{K_2(T)^{2b}}{K_2(T)} \left( \mu - (1 - \mu) \frac{A_1}{A_2} \left( \frac{K_1(T)}{K_2(T)} \right)^b \right)$$

(16)

To leapfrog Foreign, Home needs higher productivity dynamics in manufacturing than Foreign to achieve a higher productivity in that sector. From (13) and (16) we see that the advantage for Home is the higher initial productivity of technology generation two, $A_2 > A_1$, and that the disadvantage is the smaller size of its manufacturing, $L_M > L_M$. From (15), it’s also visible that the size of Home manufacturing is positively affected by the manufacturing expenditure share $\mu$, but the impact of the learning elasticity $b$ has still to be determined. In the following section, we determine, with the help of the productivity dynamics in Home and Foreign, a necessary condition for dynamic Leapfrogging that clarifies when the timing of technology shock in Leapfrogging of the advanced country through the backward country.
3.3. The Necessary Condition for Dynamic Leapfrogging

Home and Foreign productivity dynamics described by (16) and by (13), may differ and dynamics may be faster for the backward or for the advanced country. So the exclusive introduction of a new technology at Home at $T_1$ may be followed by faster dynamics in that country, but this has not to be the rule. As previously stated to start Leapfrogging a higher productivity growth in Home is necessary after $T_1$. Furthermore, the higher speed in Home productivity growth has to go on until Home manufacturing leapfrogs Foreign manufacturing at a time $T_2$. This information and the specified yield curve give us the possibility to prove the necessary condition for dynamic Leapfrogging under this specification.

The proof results from the intuition that before Leapfrogging occurs Home productivity has to achieve in $T_2'$ before $T_2$ at least parity with Foreign productivity. Leapfrogging occurs in $T_2'$, Home productivity growth is still higher than Foreign productivity growth. This has to be true, in the shown interval, where the growth differential between Home and Foreign decreases with productivities converging, so that any higher growth in $T_2'$ at Home has to be linked to higher growth at Home between $T_1$ and $T_2'$. The proof is as follows:

We assume that at $T_2'$ productivity dynamics between Home using $A_2$ and Foreign using $A_1$ are identical:

$$d \left( \frac{A_2(K_2(T_2'))}{A_1(K_1^*(T_2'))} \right) / d T_2 = 0, \quad (17)$$

Utilizing the dynamics from (16) and (13), this gives:

$$\mu \frac{A_2}{A_1} \left( \frac{K_2(T_2')}{K_1^*(T_2')} \right)^b - (1 - \mu) - \frac{K_2(T_2')}{K_1^*(T_2')} = 0 \quad (18)$$

Assume furthermore that in $T_2'$ productivity differentials between Home and Foreign have completely vanished, so that:

$$A_2 \left( K_2(T_2') \right) = A_1^* \left( K_1^*(T_2') \right), \quad \text{or} \quad \frac{A_2 \left( K_2(T_2') \right)}{A_1^* \left( K_1^*(T_2') \right)} = \frac{A_2 \left( K_2^*(T_2') \right)}{A_1 \left( K_1 (T_2) \right)} = 1 \quad (19)$$
By incorporating (19) into (18), we eliminate relative cumulated output, \( K_2(T_2^*)/K_1^*(T_2^*) \) at time \( T_2^* \). It gives:

\[
A_2 = \frac{A_1}{(2\mu - 1)^b}
\]  

(20)

From (20), it can be seen, that for any allowed value of higher manufacturing in Foreign between \( T_1 \) and \( T_2^* \), \( X < A_2 \left( K_1^*(T_2^*) \right)/A_1^* \left( K_2^*(T_2^*) \right) \) diminishes the RHS of (20), so that \( A_1 < A_1^* \left( 2\mu - 1 \right)^b \). Hence, equation (20) achieves exact productivity convergence, at any point of time after the technological shock. Given that before the alignment Home productivity dynamics were the higher than Foreign ones, Equation (20) achieves exact productivity convergence after the technology shock.

So the following inequality proves that any new technology that passes the condition:

\[
A_2 > \frac{A_1}{(2\mu - 1)^b}
\]  

(21)

and which is with (9) or (10) exclusively implementable in Home determines Leapfrogging. So there is a time \( T_2 \) when Home productivity in manufacturing can overtake Foreign productivity, \( A_2 \left( K_2^*(T_2^*) \right) > A_1^* \left( K_1^*(T_2^*) \right) \). (21) is the missing necessary dynamic condition in the literature to cause Leapfrogging under the given yield curve specification.

Comparative statics of the RHS from (21) show the positive effect of \( \mu \) and the negative effect of \( A_1 \) and \( b \) on the initial productivity requirements of a technology shock to guarantee Leapfrogging. Here the effects of an increasing \( \mu \) and a decreasing \( A_1 \) intuitive. The bigger is the demand for manufactures, \( \mu \), the bigger has to be the world manufacturing. With all its labour absorbed, the size of the manufacturing sector in Foreign is fixed and hence a bigger world manufacturing implies a bigger manufacturing at non-specialised Home. With this, output grows faster at Home and therefore productivity in the new technology grows faster, whereas competing Foreign productivity dynamics remain unchanged.

\[\text{It is shown in Chen (1999) that the lower bound } X \text{ may be different from (9) or (10), and that in the interval } \frac{A_1}{A_1^* (K_1^*(T_1^*))} \in \left(1 - \mu/|\mu X| \right) \text{ no Leapfrogging occurs. A numerical solution for } X < 1 \text{ can be found with } \frac{A_2}{A_1^* (K_1^*(T_1^*))} = \left(1/K_1^*(T_1^*) + (1 - \mu)/|\mu X| \right). \text{ This is a special case, where the initial productivity of the new technology does not matter anymore. We assume here that this ratio does not fall in this No-Leapfrogging-Interval.}\]
smaller the quality of the previous technology $A_1$, the easier it becomes to overtake with a new technology.

Less intuitive is the negative impact of the learning elasticity $b$ or the slope of learning on Leapfrogging. Here the reason comes from the assumed concave but unlimited yield curves. A higher learning elasticity grows the learning-by-doing through cumulated output. This speeds up productivity dynamics in both countries, but the effect is more accented in the more producing advanced country Home than in the not specialised backward country Foreign. So first of all a higher $b$ means higher learning in existing technologies that hampers leapfrogging.

3.4. Contribution to the Literature

The implementation of a simple and widely applicable yield curve into the Leapfrogging of Brezis, Krugman and Tsiddon (1993) improves the theory by defining precisely when Leapfrogging is going to take place. Some imprecision was left here because only the necessary static conditions for Leapfrogging have been part of their model. The presented dynamic necessary condition closes this gap.

Results for the size of the industrial sector and the effect of previous technologies are in line with Chen (1999) who presented a first alternative paper to fill that gap. In a departure from his contribution, the work presented here shows a more user-friendly derivation of the dynamic Leapfrogging condition. It also goes further in implementing the learning elasticity. The learning elasticity is important in this context because in distinction to the effect of manufactures expenditure share, it can be evaluated only in the specified model. Learning elasticity matters and as compared to general intuition, has a negative effect. This steepness of the yield curve benefits learning by doing in the backward economy but the impact is less than in the advanced economy.

4. Conclusion

The Leapfrogging model gives a simple, very intuitive story for cycles of per capita income leadership in open economies, but its dynamics are underspecified. Thus, it is not possible to explain under which condition the income leadership economy is overtaken by the initial backward economy. We closed this gap by introducing a simple yield curve in the model. The extension gives us a threshold value for the necessary dynamic condition for Leapfrogging. Therefore, the necessary improvement of the emerging new technology can be determined. A higher expenditure share for the advanced product lowers this necessary improvement and thus eases Leapfrogging. A higher learning elasticity increases it, and thus hampers Leapfrogging.
References


